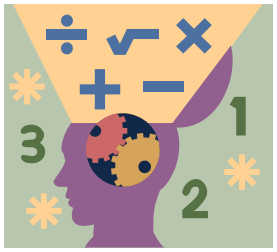
The Algebra of Functions



Operations:

Ignore the function labels. Put the right side of each equation in parentheses.

e.g. $f(x) = x^2 + 6x - 2$ $\quad$ $g(x) = x + 5$

$(x^2 + 6x - 2)$ $\quad$ $(x + 5)$

- Addition: Write those two expressions in their parentheses and put a + between
  - e.g. $(x^2 + 6x - 2) + (x + 5)$
    $x^2 + 7x + 3$ $\rightarrow$ $(f + g)(x) = x^2 + 7x + 3$
- Subtraction: Write those 2 expressions in their parentheses and put a – between
  - ** *Remember to distribute the - !*
  - e.g. $(x^2 + 6x - 2) - (x + 5)$ $\rightarrow$ $x^2 + 6x - 2 - x - 5$
    $x^2 + 5x - 7$ $\rightarrow$ $(f - g)(x) = x^2 + 5x - 7$
- Multiplication: Write those 2 expressions in their parentheses next to each other
  - ** *Remember to distribute!*
  - e.g. $(x^2 + 6x - 2)(x + 5)$ $\rightarrow$ $x^3 + 5x^2 + 6x^2 + 30x - 2x - 10$
    $x^3 + 11x^2 + 8x - 10$ $\rightarrow$ $(f + g)(x) = x^2 + 7x + 3$
- Division: Write those two expressions as a fraction. Reduce if possible.
  - e.g. $\frac{x^2 + 6x - 2}{x + 5}$ (can't reduce) $\rightarrow$ $(f / g) = \frac{x^2 + 6x - 2}{x + 5}$

Composite Functions: $f \circ g$ and $g \circ f$ also known as $f(g(x))$ and $g(f(x))$

Take the second expression and put it (in its parentheses) in place of the x in the first expression. Still using $f(x) = x^2 + 6x - 2$ and $g(x) = x + 5$:

e.g. $x^2 + 6x - 2$ $\rightarrow$ $(x + 5)^2 + 6(x + 5) - 2$

$(x + 5)$ $\quad$ $(x + 5)$ $\quad$ $f(g(x)) = x^2 + 16x + 53$

e.g. $x + 5$ $\rightarrow$ $x^2 + 6x - 2 + 5$

$(x^2 + 6x - 2)$ $\quad$ $g(f(x)) = x^2 + 6x + 3$

Inverse functions: $f^{-1}(x)$

Replace the function label – the $f(x)$ – with a y. Then switch the places of the x and y in this new equation and isolate y. Set the equation to $y =$ and replace the y with the function label $f^{-1}(x)$. ****This is not an exponent! It is another label and means "inverse function."**

e.g. $f(x) = \frac{x+3}{5} \rightarrow \quad y = \frac{x+3}{5} \rightarrow x = \frac{y+3}{5} \rightarrow 5x = y + 3 \rightarrow 5x - 3 = y$

$f^{-1}(x) = 5x - 3$